Concentration of measures via size biased couplings

Subhankar Ghosh
University of Southern California

Let Y be a nonnegative random variable with mean μ and finite positive variance σ^2, and let Y^s, defined on the same space as Y, have the Y size biased distribution, characterized by

$$E[Yf(Y)] = \mu E f(Y^s) \quad \text{for all functions } f \text{ for which these expectations exist.}$$

Under a variety of conditions on Y and the coupling of Y and Y^s, including combinations of boundedness and monotonicity, one sided concentration of measure inequalities such as

$$P\left(\frac{Y-\mu}{\sigma} \geq t\right) \leq \exp\left(-\frac{t^2}{2(A+Bt)}\right) \quad \text{for all } t \geq 0$$

hold for some explicit A and B. The theorem can be applied to a number of situations the number of bulbs switched on at the terminal time in the so called lightbulb process of Rao, Rao, and Zhang, the number of nonisolated balls in a coverage process, the number of relatively ordered subsequences in a uniform random permutation etc.

Similar concentration results can be obtained for some specific unbounded couplings as well, for example number of isolated vertices in the Erdös-Rényi random graph model and generalised variance for multivariate normal random samples.

Recently, some more similar concentration results were obtained for coordinates of a random vector $\mathbf{W} \in \mathbb{R}^k$, for which one can obtain an exchangeable tuple $(\mathbf{W}, \mathbf{W}')$ satisfying the condition

$$E(\mathbf{W}'|\mathbf{W}) = (I_k - \Lambda)\mathbf{W} + \mathbf{R}(\mathbf{W}).$$

Some examples include complete nondegenerate U statistics and doubly indexed permutation statistic.

Part of the above is joint work with Larry Goldstein.