

Homework 10 Supplement, Statistics 200A Fall 2011
Adapted from “The Essentials of Probability Theory” by Rick
Durrett

1. Let Z_1 and Z_2 independent random variables with finite variances. Let $X = Z_1$ and $Y = Z_1 + Z_2$. Find the correlation of X and Y and the best linear predictor $aX + b$ of Y .
2. Let X_1, X_2, \dots be independent with $\text{Var}(X_i) = \sigma^2$ for $i = 1, 2, \dots$. What is the correlation of $\sum_{i=1}^k X_i$ and $\sum_{i=1}^n X_i$ if $k \leq n$?
3. Let X_n be the amount of sewage dumped into the bay on day n and suppose that some fraction $1 - p$ gets carried out to sea each day. Assume that the X_i are independent and let

$$Y_n = \sum_{m=0}^n p^m X_{n-m}$$

be the amount of sewage in the bay on day n according this model.

- (a) What is $\mathbb{E}[Y_i]$?
 - (b) What is $\text{Var}(Y_i)$?
 - (c) What is the correlation between Y_0 and Y_n . Note that $Y_n = p^n Y_0 + Z$ where Z is independent of Y_0 .
4. Let U be uniform on the interval $(0, 1)$ and $Y|X = x$ be uniform on the interval $(0, x)$.
 - (a) What is $\mathbb{E}[Y|U]$?
 - (b) What is $\text{Var}(Y|U)$?
 - (c) What is $\mathbb{E}[Y]$?
 - (d) What is $\text{Var}(Y)$?
 5. Let X_1, X_2, \dots, X_n be independent with the same distribution and $m \leq n$. What is $\mathbb{E}[X_1 + \dots + X_m | X_1 + \dots + X_n]$?
 6. Let $X \sim \text{Bi}(n, p)$ and $Y \sim \text{Bi}(m, p)$ independent. What is $\mathbb{E}[X|X + Y]$?
 7. Let $X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\mu)$ independent. What is $\mathbb{E}[X|X + Y]$ and $\text{Var}(X|X + Y)$?
 8. Let (X, Y) have joint density $e^{-x}, 0 < y < x$. Find $\mathbb{E}[X|Y]$ and $\text{Var}(X|Y)$.