

Homework 11 Supplement, Statistics 200A Fall 2010  
Adapted from “The Essentials of Probability Theory” by Rick  
Durrett

1. Let  $Z_1$  and  $Z_2$  independent random variables with finite variances. Let  $X = Z_1$  and  $Y = Z_1 + Z_2$ . Find the correlation of  $X$  and  $Y$  and the best linear predictor  $aX + b$  of  $Y$ .
2. Let  $X_1, X_2, \dots$  be independent with  $\text{Var}(X_i) = \sigma^2$  for  $i = 1, 2, \dots$ . What is the correlation of  $\sum_{i=1}^k X_i$  and  $\sum_{i=1}^n X_i$  if  $k \leq n$ ?
3. Let  $X_n$  be the amount of sewage dumped into the bay on day  $n$  and suppose that some fraction  $1 - p$  gets carried out to sea each day. Assume that the  $X_i$  are independent and let

$$Y_n = \sum_{m=0}^n p^m X_{n-m}$$

be the amount of sewage in the bay on day  $n$  according this model.

- (a) What is  $\mathbb{E}[Y_i]$ ?
  - (b) What is  $\text{Var}(Y_i)$ ?
  - (c) What is the correlation between  $Y_0$  and  $Y_n$ . Note that  $Y_n = p^n Y_0 + Z$  where  $Z$  is independent of  $Y_0$ .
4. Let  $U$  be uniform on the interval  $(0, 1)$  and  $Y|U = x$  be uniform on the interval  $(0, x)$ .
    - (a) What is  $\mathbb{E}[Y|U]$ ?
    - (b) What is  $\text{Var}(Y|U)$ ?
    - (c) What is  $\mathbb{E}[Y]$ ?
    - (d) What is  $\text{Var}(Y)$ ?
  5. Let  $X_1, X_2, \dots, X_n$  be independent with the same distribution and  $m \leq n$ . What is  $\mathbb{E}[X_1 + \dots + X_m | X_1 + \dots + X_n]$ ?
  6. Let  $X \sim \text{Bi}(n, p)$  and  $Y \sim \text{Bi}(m, p)$  independent. What is  $\mathbb{E}[X|X + Y]$ ?
  7. Let  $X \sim \text{Po}(\lambda)$  and  $Y \sim \text{Po}(\mu)$  independent. What is  $\mathbb{E}[X|X + Y]$  and  $\text{Var}(X|X + Y)$ ?
  8. Let  $(X, Y)$  have joint density  $e^{-x}, 0 < y < x$ . Find  $\mathbb{E}[X|Y]$  and  $\text{Var}(X|Y)$ .