Homework 2 Supplement, Statistics 200A, Fall 2011

Let S be a finite or countable set; that is, $S = \{s_1, \ldots, s_n\}$ for some $n \geq 1$ or $S = \{s_1, s_2, \ldots\}$.

1. Let P be a function on S such that

$$P(s_i) \geq 0 \text{ for all } i, \quad \text{and} \quad \sum_{s \in S} P(s) = 1. \tag{1}$$

   If we extend P to be a function on subsets A of S by

$$P(A) = \sum_{a \in A} P(a), \tag{2}$$

   then show that S with probability function P is a probability space.

2. Alternatively, if Q is some probability function on S (countable as above), show that (1) and (2) are satisfied (with P replaced by Q), where we interpret $Q(s_i) = Q(\{s_i\})$.

These two exercises imply that in order to define a probability function on a finite or countable (also called discrete) outcome space, it is enough to define the function on singletons $\{s\}$, such that it satisfies (1). From this point, the probability function on general sets is given by (2).