Selected Problems from Pitman's "Probability" Text

Statistics 200A, Nathan Ross, Fall 2010

1.6.6

Suppose you roll a fair six-sided die repeatedly until the first time you roll a number that you have rolled before.

1. For each $r = 1, 2, \dots$ calculate the probability p_r that you roll exactly r times.
2. Without calculation, write down the value of $p_1 + p_2 + \cdots + p_{10}$. Explain.
3. Check that your calculated values of p_r have this value for their sum.

1.r.11

A hat contains n coins, f of which are fair, and b of which are biased to land heads with probability 2/3. A coin is drawn from the hat and tossed twice. The first time it lands heads, and the second time it lands tails. Given this information, what is the probability that it is a fair coin?

3.4.8

In the game of *craps* a player throws two dice and observes the sum. A throw of 7 or 11 is an immediate win. A throw of 2, 3, or 12 is an immediate loss. A throw of 4, 5, 6, 8, 9, or 10 becomes the player's *point.* In order to win the game now, the player must continue to throw the dice, and obtain the point before throwing a 7. The problem is to calculate the probability of winning at craps. For $i = 2, \dots, 12$, let A_i, be the event that the first sum thrown is equal to i.

1. Show that for $i = 4, 5, 6, 8, 9, 10$,
$$P(\text{Win}|A_i) = P(A_i)/[P(A_i) + P(A_7)].$$
2. Write down $P(\text{Win}|A_i)$ for the other possible values i.
3. Deduce that the probability of winning at craps is
$$P(\text{Win}) = \frac{1952}{36 \times 11 \times 10} = 0.493\dots$$

3.4.11

Suppose that A tosses a coin which lands heads with probability p_A, and B tosses one which lands heads with probability p_B. They toss their coins simultaneously over and over again, in a competition to see who gets the first head. The one to get the first head is the winner, except that a draw results if they get their first heads together. Calculate:

1. $P(\text{A wins})$;
2. $P(\text{B wins})$;
3. $P(\text{draw})$;
4. the distribution of the number of times A and B must toss.