Selected Problems from Pitman's "Probability" Text

Statistics 200A, Nathan Ross, Fall 2009

3.4.8

In this game a player throws two dice and observes the sum. A throw of 7 or 11 is an immediate win. A throw of 2, 3, or 12 is an immediate loss. A throw of 4, 5, 6, 8, 9, or 10 becomes the player's *point.* In order to win the game now, the player must continue to throw the dice, and obtain the point before throwing a 7. The problem is to calculate the probability of winning at craps. Let X_0 represent the first sum thrown. The basic idea of the calculation is first to calculate $P(\text{Win} \mid X_0 = x)$ for every possible value x of X_0, then use the law of average conditional probabilities to obtain $P(\text{Win})$.

1. Show that for $x = 4, 5, 6, 8, 9, 10$,

$$P(\text{Win} \mid X_0 = x) = P(x)/[P(x) + P(7)]$$

   where $P(x) = P(X_i = x)$ is the probability of rolling a sum of x. (Refer to Example 2).

2. Write down $P(\text{Win} \mid X_0 = x)$ for the other possible values x of X_0.
3. Deduce that the probability of winning at craps is

$$P(\text{Win}) = \frac{1952}{36 \times 11 \times 10} = 0.493\ldots$$

3.4.11

Suppose that A tosses a coin which lands heads with probability p_A, and B tosses one which lands heads with probability p_B. They toss their coins simultaneously over and over again, in a competition to see who gets the first head. The one to get the first head is the winner, except that a draw results if they get their first heads together. Calculate:

1. $P(\text{A wins})$;
2. $P(\text{B wins})$;
3. $P(\text{draw})$;
4. the distribution of the number of times A and B must toss.

3.4.15

Suppose F has geometric distribution on $\{0, 1, 2, \ldots\}$.

1. Show that for every $k \geq 0$,

$$P(F - k = m \mid F \geq k) = P(F = m), \quad m = 0, 1, \ldots$$

2. (Optional; requires induction) Show the geometric distribution is the only discrete distribution on $\{0, 1, 2, \ldots\}$ with this property.