Selected Problems from Pitman's "Probability" Text

Statistics 200A, Nathan Ross, Fall 2009

3.2.6 Let X be the number of spades in 7 cards dealt from a well-shuffled deck of 52 cards containing 13 spades. Find $E(X)$.

3.2.7 In a circuit containing n switches, the ith switch is closed with probability p_i, $i = 1, \dots, n$. Let X be the total number of switches that are closed. What is $E(X)$? Or is it impossible to say without further assumptions?

3.2.8 Suppose $E(X^2) = 3, \quad E(Y^2) = 4, \quad E(XY) = 5$. Find $E\left[(X+Y)^2\right]$.

3.2.10 Let A and B be independent events, with indicator random variables I_A and I_B. What is $E(I_A + I_B)^2$?

3.2.14 A building has 10 floors above the basement. If 12 people get into an elevator at the basement, and each chooses a floor at random to get out, independently of the others, at how many floors do you expect the elevator to make a stop to let out one or more of these 12 people?

3.2.16 A standard deck of 52 cards is shuffled and dealt. Let X_1 be the number of cards appearing before the first ace, X_2 the number of cards between the first and second ace (not counting either ace), X_3 the number between the second and third ace, X_4 the number between the third and fourth ace, and X_5 the number after the last ace. It can be shown that each of these random variables X_i has the same distribution, $i = 1, 2, \dots, 5$, and you can assume this to be true.

1. Write down a formula for $P(X_i = k)$, $0 \leq k \leq 48$.
2. Show that $E(X_i) = 9.6$. [*Hint*: Do not use your answer to a.]
3. Are $X_1, \dots, X_5$ pairwise independent? Prove your answer.

3.2.20 Show that the distribution of a random variable X with possible values 0, 1, and 2 is determined by $\mu_1 = E(X)$ and $\mu_2 = E(X^2)$, by finding a formula for $P(X = x)$ in terms of μ_1 and μ_2, $\quad x = 0, 1, 2$.

3.2.22 Consider a sequence of $n \geq 4$ independent trials, each resulting in success (S) with probability p, and failure (F) with probability $1-p$. Say a *run of three successes* occurs at the beginning of the sequence if the first four trials result in SSSF; a run of three successes occurs at the end of the sequence if the last four trials result in FSSS; and a run of three successes elsewhere in the sequence is the pattern FSSSF. Let $R_{3,n}$ denote the number of runs of three successes in the n trials.

1. Find $E(R_{3,n})$.
2. Define $R_{m,n}$, the number of success runs of length m in n trials, similarly for $1 \leq m \leq n$. Find $E(R_{m,n})$.
3. Let R_n be the total number of non-overlapping success runs in n trials, counting runs of any length between 1 and n. Find $E(R_n)$ by using the result of b).
4. Find $E(R_n)$ another way by considering for each $1 \leq j \leq n$ the number of runs that start on the jth trial. Check that the two methods give the same answer.