Homework 6 Supplement, Statistics 200A Fall 2011

1. Let F_1 and F_2 be two strictly increasing continuous distribution functions such that $F_1(x) \leq F_2(x)$ for all $x \in \mathbb{R}$. Show that there are random variables X_1 and X_2 with these respective distribution functions such that $X_1 \geq X_2$ (more precisely, $\mathbb{P}(X_1 \geq X_2) = 1$). Hint: Use the construction of a random variable from a uniform random variable.

2. Assume a dart is thrown randomly at a dartboard of radius one unit. Let (X, Y) be the position (in cartesian coordinates) of the dart if the bulls-eye is placed at the origin. Assume that for some constant C, the density of (X, Y) is

$$f(x, y) = C(x^2 + y^2)^{-1/2}, \quad x^2 + y^2 \leq 1.$$

   Let R be the distance to the origin and Θ the angle with the horizontal of the point (X, Y).

   (a) What is the constant C?

   (b) What is the joint density of (R, Θ)?

   (c) What is the (marginal) density of R?

   (d) What is the (marginal) density of Θ?

   (e) Are R and Θ independent?

3. Let U_1 and U_2 independent uniform random variables on $(0, 1)$ and define $M_1 = \min\{U_1, U_2\}$ and $M_2 = \max\{U_1, U_2\}$.

   (a) What is the joint density of (M_1, M_2)?

   (b) What is the (marginal) density of M_1?

   (c) What is the (marginal) density of M_2?

   (d) Are M_1 and M_2 independent?

   (e) What is the chance that $M_1 = U_1$?

   (f) What is the chance that $M_2 = U_1$?

4. Let X_1 and X_2 be independent standard normal random variables.

   (a) What is the p.d.f. of $X_1^2 + X_2^2$?

   (b) What is the p.d.f. of $\sqrt{X_1^2 + X_2^2}$?