

Homework 7 Supplement, Statistics 200A Fall 2010

1. Assume a dart is thrown randomly at a dartboard of radius one unit. Let (X, Y) be the position (in cartesian coordinates) of the dart if the bulls-eye is placed at the origin. Assume that for some constant C , the density of (X, Y) is

$$f(x, y) = C(x^2 + y^2)^{-1/2}, \quad x^2 + y^2 \leq 1.$$

Let R be the distance to the origin and Θ the angle with the horizontal of the point (X, Y) .

- (a) What is the constant C ?
 - (b) What is the joint density of (R, Θ) ?
 - (c) What is the (marginal) density of R ?
 - (d) What is the (marginal) density of Θ ?
 - (e) Are R and Θ independent?
2. Let U_1 and U_2 independent uniform random variables on $(0, 1)$ and define $M_1 = \min\{U_1, U_2\}$ and $M_2 = \max\{U_1, U_2\}$.
 - (a) What is the joint density of (M_1, M_2) ?
 - (b) What is the (marginal) density of M_1 ?
 - (c) What is the (marginal) density of M_2 ?
 - (d) Are M_1 and M_2 independent?
 - (e) What is the chance that $M_1 = U_1$?
 - (f) What is the chance that $M_2 = U_1$?
 3. Let X_1 and X_2 be independent standard normal random variables.
 - (a) What is the p.d.f. of $X_1^2 + X_2^2$?
 - (b) What is the p.d.f. of $\sqrt{X_1^2 + X_2^2}$?