Homework 8 Supplement, Statistics 200A Fall 2010

1. Show that if $\mathbf{X} \sim \mathrm{MN}\left(\boldsymbol{\mu}, \Sigma\right)$ is multivariate normal with $(\Sigma)_{i,i} = \sigma_i^2 > 0$ for $i = 1, \ldots, n$, and $(\Sigma)_{i,j} = 0$ for $i \neq j$, then $\{X_1, \ldots, X_n\}$ are independent.

2. Let $Y \sim \mathrm{Beta}(a, b)$, with a and b positive integers; i.e. Y has p.d.f.

$$f_Y(y) = \frac{y^{a-1}(1-y)^{b-1}}{B(a,b)}, \qquad 0 < y < 1,$$

   where $B(a, b)$ is the beta function. Let $(X|Y = y) \sim \mathrm{Bi}(n, y)$.

   (a) What is the joint p.d.f of (X, Y)?

   (b) What is the marginal probability function of X?

   (c) What is the p.d.f. of $Y|X = x$?

3. Let $N \sim \mathrm{Poi}(\lambda)$ with $\lambda > 0$; i.e. N has probability function

$$f_N(k) = e^{-\lambda}\frac{\lambda^k}{k!}, \qquad k = 0, 1, \ldots$$

   Let $(X|N = n) \sim \mathrm{Bi}(n, p)$ with $0 \le p \le 1$ and let $Y = N - X$.

   (a) Show that $X \sim \mathrm{Poi}(p\lambda)$. You may use the identity:

$$e^{-a}\sum_{n=k}^{\infty}\binom{n}{k}\frac{a^n}{n!} = \frac{a^k}{k!}, \qquad a \in \mathbb{R}.$$

   (b) Show that $Y \sim \mathrm{Poi}((1-p)\lambda)$.

   (c) Show that X and Y are independent.