Homework 8 Supplement, Statistics 200A Fall 2011

1. Show that if $\mathbf{X} \sim \mathrm{MN}(\boldsymbol{\mu}, \Sigma)$ is multivariate normal with $(\Sigma)_{i,i} = \sigma_i^2 > 0$ for $i = 1, \ldots, n$, and $(\Sigma)_{i,j} = 0$ for $i \neq j$, then $\{X_1, \ldots, X_n\}$ are independent.

2. Let $\mathbf{X} := (X_1, X_2) \sim \mathrm{MN}(\boldsymbol{\mu}, \Sigma)$ be multivariate normal having the decomposition $\mathbf{X} = R\mathbf{Z} + \boldsymbol{\mu}$, where $\mathbf{Z}$ is a vector of two independent standard normals, and

$$R = \begin{bmatrix} \sigma_1 & 0 \\ \sigma_2\rho & \sigma_2\sqrt{1-\rho^2} \end{bmatrix},$$

   for some $\sigma_1, \sigma_2 > 0$ and $-1 \leq \rho \leq 1$. Show using densities that $X_2 | X_1 = x_1$ is a normal distribution and determine its parameters.

3. Let $X_1, \ldots, X_n$ be independent Poisson random variables having respective parameters $\lambda_1, \ldots, \lambda_n$. Show that $\sum_{i=1}^n X_i$ is a Poisson random variable and determine its parameter. You should do small cases (e.g. $n = 2$) first.