

## Homework 9 Supplement, Statistics 200A Fall 2010

1. Let  $\mathbf{X} := (X_1, X_2) \sim \text{MN}(\boldsymbol{\mu}, \Sigma)$  be multivariate normal having the decomposition  $\mathbf{X} = R\mathbf{Z} + \boldsymbol{\mu}$ , where  $\mathbf{Z}$  is a vector of two independent standard normals, and

$$R = \begin{bmatrix} \sigma_1 & 0 \\ \sigma_2\rho & \sigma_2\sqrt{1-\rho^2} \end{bmatrix},$$

for some  $\sigma_1, \sigma_2 > 0$  and  $-1 \leq \rho \leq 1$ . Show using densities that  $X_2|X_1 = x_1$  is a normal distribution and determine its parameters.

2. Let  $X_1, \dots, X_n$  be independent Poisson random variables having respective parameters  $\lambda_1, \dots, \lambda_n$ . Show that  $\sum_{i=1}^n X_i$  is a Poisson random variable and determine its parameter. You should do small cases (e.g.  $n = 2$ ) first.
3. Let  $X \sim \text{Exp}(\lambda)$ . Find  $\mathbb{E}[X]$  and  $\mathbb{E}[X^2]$ .
4. Let  $X \sim \text{Po}(\lambda)$ . Find  $\mathbb{E}[X]$  and  $\mathbb{E}[X^2]$ .