

Selected Problems from Pitman's "Probability" Text

Statistics 200A, Nathan Ross, Fall 2009

2.r.18

Seven dice are rolled. Write down unsimplified expressions for the probabilities of each of the following events:

1. exactly three sixes;
2. three of one kind and four of another;
3. two fours, two fives, and three sixes;
4. each number appears;
5. the sum of the dice is 9 or more.

2.r.23

Ten percent of the families in a town have no children, twenty percent have one child, forty percent have two children, twenty percent have three, and ten percent have four. Assume each child in a family is equally likely to be a boy or a girl, independently of all the others. A family is picked at random from this town. Given that there is at least one boy in the family, what is the chance that there is also at least one girl?

3.4.12

Let W_1 and W_2 be independent geometric random variables with parameters p_1 and p_2 . Find:

1. $P(W_1 = W_2)$;
2. $P(W_1 < W_2)$;
3. $P(W_1 > W_2)$;
4. the distribution of $\min(W_1, W_2)$;
5. the distribution of $\max(W_1, W_2)$.

3.4.13

Consider the following gambling game for two players, Black and White. Black puts b black balls and White puts w white balls in a box. Black and White take turns at drawing at random from the box, with replacement between draws until either Black wins by drawing a black ball or White wins by drawing a white ball. Suppose Black gets to draw first.

1. Calculate $P(\text{Black wins})$ and $P(\text{White wins})$ in terms of $p = b/(b + w)$.

2. What value of p would make the game fair (equal chances of winning)?
3. Is the game ever fair?
4. What is the least total number of balls in the game, $(b + w)$, such that neither player has more than a 51% chance of winning?

3.5.17

Assume that during a storm the number of raindrops that fall on any given square inch of ground during any 10 second period is a Poisson random variable with parameter five.

1. What is the chance that a particular square inch is not hit by any drops during a given 10-second period?
2. If each drop is a big drop with probability $2/3$ and a small drop with probability $1/3$, independently of the other drops, what is the chance that during 10 seconds a particular square inch gets hit by precisely four big drops and five small ones?

3.r.2

A fair die is rolled repeatedly. Calculate

1. the chance that the first 6 appears before the tenth roll;
2. the chance that the third 6 appears on the tenth roll.

3.r.3

Two fair dice are rolled independently. Let X be the maximum of the two rolls, and Y the minimum.

1. What is $P(X = x)$ for $x = 1, \dots, 6$?
2. What is $P(Y = y|X = 3)$ for $y = 1, \dots, 6$?
3. What is the joint distribution of X and Y ?

3.r.19

Suppose that X has Poisson(μ) distribution, and that Y has geometric(p) distribution on $\{0, 1, 2, \dots\}$ independently of X .

1. Find a formula for $P(Y \geq X)$ in terms of p and μ .
2. Evaluate numerically for $p = 1/2$ and $\mu = 1$.