

MAST30001 2013, Recommended Problems, Chapter 3

Lecturer: Nathan Ross

Instructions: Answer the following questions. Clearly show all work and give clear and concise explanations, using prose when appropriate. Clarity, neatness and style count.

1. A simplified model for the spread of a contagion in a small population of size 4 is as follows. At each discrete time unit, two individuals in the population are chosen uniformly at random to meet. If one of these persons is healthy and the other has the contagion, then with probability $1/4$ the healthy person becomes sick. Otherwise the system stays the same.
 - (a) If X_n is the number of healthy people at step n , then explain why $X_0, X_1 \dots$ is a Markov chain.
 - (b) Specify the transition probabilities of X_n .
 - (c) If initially the chance that a given person in the population has the disease equals $1/2$, determined independently, then what is the chance everyone has the disease after two steps in the process?
 - (d) Assume now that the process begins with exactly one person infected. Given that not everyone is infected after three steps of the process, what is the chance exactly one person is infected?