MAST30001 2013, Recommended Problems, Chapter 7
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Instructions: Answer the following questions. Justify all work and give clear, concise explanations, using prose when appropriate. Clarity, neatness and style count.

1. (M/M/∞ queue) Assume that in a queuing system customers arrive according to a rate λ Poisson process, customers are always served immediately (for example, customers making purchases on the internet), and the service time of a customer is exponential with rate μ, independent of arrival times and other service times.

   (a) Model this queue as a birth-death chain and write down its generator.

   (b) Describe the long run behaviour of the chain.

   (c) When the queue is in stationary (i.e., after its been running a long time), what is the expected number of customers in the system, number of customers in the queue, number of busy servers, and service time for an arriving customer?

   (d) Let X_t be the number of customers in the system (including those being served) at time t and set $X_0 = 0$. What is $\mathbb{E}X_t$? *[Hint: if $m(t) = \mathbb{E}X_t$, consider $m'(t)$.]* You should check your formula makes sense as t tends to infinity.

2. (M/G/∞ queue) In a certain communications system, information packets arrive according to a Poisson process with rate λ per second and each packet is processed in one second with probability p and in two seconds with probability $1-p$, independent of the arrival times and other service times. Let N_t be the number of packets that have entered the system up to time t and X_t be the number of packets in the system (including those being served) at time t.

   (a) Is $(X_t)_{t\geq 0}$ a Markov chain?

   (b) If $X_0 = 0$, what is the distribution of X_2?

   (c) If $X_0 = 0$, is there a "stationary" limiting distribution $\pi_k = \lim_{t\to\infty} \mathbb{P}(X_t = k)$? If so, what is it?

   (d) If $X_0 = N_0 = 0$, what is the joint distribution of X_t and N_t?