Source-Based Jamming for Physical-Layer Security on Untrusted Full-Duplex Relay

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***Abstract*—We address the problem of secure wireless communications over an untrusted full-duplex (FD) relay based on the source jamming scheme. The optimal power allocation between the confidential signal and the jamming signal is derived to maximize the secrecy rate. Then the corresponding secrecy outage probability (SOP) and the average secrecy rate (ASR) are analyzed. A tight approximation and an asymptotic result are further obtained for the single-antenna destination case both in simple forms. The large-antenna destination case is also analyzed rigorously. Further discussion reveals that transmit-power dependent self-interference has significant negative impact on the secrecy performance.**

***Index Terms*—Full-duplex, optimal power allocation, physical layer security, source jamming, untrusted relay.**

I. Introduction

The rise of smart devices and Internet of Things (IoT) has also coincided with a rise in potential attacks from eavesdroppers. Physical-layer security (PLS) helps to some extent to protect wireless systems from such attacks [1]. For an untrusted cooperative network, the relay which is generally implemented with the amplify-and-forward (AF) protocol operates either in half-duplex (HD) or full-duplex (FD) mode. When the residual self-interference (RSI) is suppressed to a very low level, the FD mode achieves better multiplexing gain. Thus, the PLS of AF-FD untrusted relaying is the focus of this paper. While the PLS can be enhanced by proper resource allocation [2]–[4], signal jamming is an alternative technique to further improve the security. Depending on the application, the jamming signal, e.g., artificial Gaussian noise, is generated either from the source, the destination or an external friendly node [1], [5], [6]. For an HD untrusted relaying, PLS has been extensively investigated in the literature. For example, the cooperative jamming is introduced in [1] where the secrecy rate is analyzed from the information theoretic point of view. For the destination jamming, the ergodic secrecy capacity is derived in [7]. Recently, an FD destination jamming scheme with optimal antenna selection is proposed in [8] where the secrecy rate is derived with approximate expressions. However, when the relay operates in FD mode, FD destination jamming may not be possible under some circumstances due to limited resources, e.g., low-power IoT and sensors. Then, the source jamming (SBJ) where a jamming signal is embedded with the intended signals of the source, is a promising scheme [9].

In this paper, we consider the SBJ scheme for an FD-AF relay network. First, we derive the optimal power allocation between the confidential and jamming signals with respect to the secrecy capacity, and then analyze the secrecy outage probability (SOP) and the average secrecy rate (ASR) of the network achieved by the optimal power allocation. Further, we provide insightful approximations and asymptotic results of the system performance.

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II. System Model

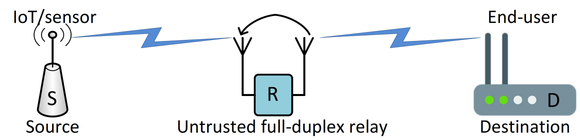


Fig. 1: A full-duplex untrusted relay network.

Since the applications of FD relaying have been widely investigated recently and malicious attack by untrusted nodes is becoming an increasing concern in 5G candidate applications, e.g., uplink/downlink transmissions in sensor and cellular networks [9], [10], we consider the SBJ scheme under FD to enhance the system's secrecy performance. As shown in Fig. 1, a single-antenna source S communicates via an untrusted FD-AF relay R having one pair of transmit-receive antenna. Its destination D has N multiple antennas. Channel gains of the $\mathrm{S}-\mathrm{R}$ link and the ℓth antenna of the $\mathrm{R}-\mathrm{D}$ link are denoted h_{sr} and h_{rd_ℓ}, $\ell \in \{1,\cdots,N\}$, respectively. The channels are assumed to be independent complex Gaussian where $h_{sr} \sim \mathcal{CN}(0,\sigma_{sr}^2)$ and $h_{rd_\ell} \sim \mathcal{CN}(0,\sigma_{rd}^2)$. With this we have assumed that h_{rd_ℓ}'s $\forall \ell$ have identical distribution, but h_{sr} and h_{rd_ℓ} are not necessary identical. The direct $\mathrm{S}-\mathrm{D}$ link is not available due to obstacles and shadowing.

The source transmits a composite signal containing the confidential signal x_s with power aP_s and the jamming signal x_j with power $(1-a)P_s$ to R, where $a \in [0,1]$ is the power allocation ratio. Both signals x_s and x_j are with unit average energy. Power budgets are fixed as P_s and P_r for S and R, respectively. We assume that D has full knowledge of x_j, and $\mathrm{S}-\mathrm{R}$ and $\mathrm{R}-\mathrm{D}$ channel state information (CSI) which helps the jamming signal cancellation, maximum-ratio combining and detection. The CSI can be obtained through a channel estimation and CSI exchange procedure before data transmission, for which the untrusted relay does not harm the training signals [11]. At time τ, the received signal at R is $y_r(\tau) = \left[\sqrt{aP_s}x_s(\tau) + \sqrt{(1-a)P_s}x_j(\tau)\right] h_{sr} + i(\tau) + n_r(\tau)$, where $n_r(\tau)$ is the noise at R with variance σ_r^2 and $i(\tau)$ is the receive residual self-interference (RSI) resulting from several stages of cancellation. The RSI is assumed to be independent to other signals and follows Gaussian distribution with zero-mean and σ_i^2-variance. The variance may be modeled as $\sigma_i^2 = \omega P_r^\nu$ where the two constants, $\omega > 0$ and $\nu \in [0,1]$, depend on the interference cancellation scheme at R [12]. The transmit signal from R is $G y_r(\tau-\delta)$ where $G \triangleq \sqrt{P_r/(P_s|h_{sr}|^2+\sigma_i^2+\sigma_n^2)}$ is the amplification coefficient and δ is the processing delay [13]. The receive signal at the ℓth antenna of D at time τ is

$y_{d_\ell}(\tau) = G y_r(\tau-\delta) h_{rd_\ell} + n_{d_\ell}(\tau)$, where $n_{d_\ell}(\tau)$ is the noise at the ℓth antenna with variance σ_d^2.

Let $\Gamma_i \triangleq \sigma_i^2/\sigma_r^2$. The signal-to-interference-plus-noise ratios (SINRs) at R and D can then be given, respectively, as

$$\Gamma_{\rm R}(a) = \frac{a\Gamma_{sr}}{(1-a)\Gamma_{sr}+1} \text{ and } \Gamma_{\rm D}(a) = \frac{a\Gamma_{sr}\Gamma_{rd}}{\Gamma_{sr}+\Gamma_{rd}+1}. \quad (1)$$

Here Γ_{sr} and Γ_{rd} are the instantaneous signal-to-noise ratios (SNRs) of $\rm S-R$ and $\rm R-D$ links, respectively, given as

$$\Gamma_{sr} \triangleq \frac{P_s}{\sigma_r^2(1+\Gamma_i)}|h_{sr}|^2 \text{ and } \Gamma_{rd} \triangleq \frac{P_r}{\sigma_d^2}\sum_{\ell=1}^{N}|h_{rd_\ell}|^2. \quad (2)$$

The instantaneous secrecy rate can then be expressed as

$$C_s(a) = \left[\ln(1+\Gamma_D(a)) - \ln(1+\Gamma_R(a))\right]^+, \quad (3)$$

where $[x]^+ \triangleq \max\{0,x\}$ and $C_s(a)$ is in [nats/sec/Hz].

III. Optimal Power Allocation

We set the power allocation factor a in order to maximize the secrecy rate. The optimal value for a can be evaluated as

$$a^* = \arg\max_{0\le a\le 1} C_s(a) = \arg\max_{0\le a\le 1} \max\{\Psi(a),1\}, \quad (4)$$

where $\Psi(a) \triangleq (1+\Gamma_D(a))/(1+\Gamma_R(a))$ and the second equality is due to the monotonicity of the logarithm function. By noting that $\partial^2\Psi(a)/\partial a^2 < 0$, we can find the unique solution for a by solving the equation $\partial\Psi(a)/\partial a = 0$ for a. Since $a \in [0,1]$, the optimal power allocation factor a^* or the transmission policy can be given as follows.

- When $\Gamma_{rd} < 1 + 1/\Gamma_{sr}$, or equivalently,

$$\frac{P_r}{\sigma_d^2}\sum_{\ell=1}^{N}|h_{rd_\ell}|^2 < 1 + \frac{\sigma_i^2+\sigma_r^2}{P_s|h_{sr}|^2}, \quad (5)$$

  $C_s(a) = 0$ for any $a \in [0,1]$. Thus the source should be kept in the idle mode to save energy.
- When (5) does not hold, the optimal value is

$$a^* = \frac{1}{2}\left(1 - \frac{1+\Gamma_{sr}}{\Gamma_{sr}\Gamma_{rd}}\right). \quad (6)$$

With the optimal power allocation factor, we have

$$\Gamma_{\rm R}^* = \frac{\Gamma_{sr}(\Gamma_{rd}-1)-1}{(\Gamma_{sr}+2)\,\Gamma_{rd}+\Gamma_{sr}+1},\ \Gamma_{\rm D}^* = \frac{\Gamma_{sr}(\Gamma_{rd}-1)-1}{2\,(\Gamma_{rd}+\Gamma_{sr}+1)}, \quad (7)$$

where $\Gamma_{\rm R}^* = \Gamma_R(a^*)$ and $\Gamma_{\rm D}^* = \Gamma_D(a^*)$. The case of no transmission, $\Gamma_{rd} < 1 + 1/\Gamma_{sr}$, happens when the $\rm S-R$ channel or all the $\rm R-D$ channels are in deep fading. The probability of this case decreases when the transmit power of either the source or the relay increases. Also, it can be easily seen that $\Gamma_{\rm R}^* < 1$ regardless of the channel realizations, meaning that the relay has very low achievable rate. On the other hand, the receive SINR of the destination does not have an upper bound, meaning that with preferable channel realizations, the destination can achieve high secrecy rate. $\Gamma_{\rm R}^*$ and $\Gamma_{\rm D}^*$ have the same numerator; but the denominators are quadratic and linear, respectively, in the link SNRs when γ_i is fixed. Further, we have $\Gamma_{\rm D}^* - \Gamma_{\rm R}^* \ge 0$, which implies that the secure information transmission is possible.

IV. Performance Analysis

For a random variable (rv) X, we denote its probability density function (p.d.f.), cumulative distribution function (c.d.f.) and complementary c.d.f. (c.c.d.f.) as $f_X(t)$, $F_X(t)$ and $\bar{F}_X(t)$, respectively. The exponential rv X with rate parameter c is denoted as $X \sim \mathcal{E}xp(c)$ where $f_X(t) = c\,e^{-c\,t}$ and $F_X(t) = 1 - e^{-c\,t}$. Since the channels are independent Rayleigh fading, from (2), it can be shown that

$$\Gamma_{sr} \sim \mathcal{E}xp\left(1/\bar{\gamma}_r\right) \text{ where } \bar{\gamma}_r \triangleq P_s\sigma_{sr}^2/\sigma_r^2(\Gamma_i+1). \quad (8)$$

$\bar{\gamma}_r$ represents the average SINR of the $\rm S-R$ link. Further from (2), Γ_{rd} is a sum of N i.i.d. exponential rvs whose p.d.f. and c.d.f. are, respectively, for $t \ge 0$

$$F_{\Gamma_{rd}}(t) = 1 - \sum_{k=0}^{N-1}\frac{t^k e^{-t/\bar{\gamma}_d}}{k!\bar{\gamma}_d^k} \text{ and } f_{\Gamma_{rd}}(t) = \frac{t^{N-1}e^{-t/\bar{\gamma}_d}}{(N-1)!\bar{\gamma}_d^N}, \quad (9)$$

where $\bar{\gamma}_d \triangleq P_r\sigma_{rd}^2/\sigma_d^2$ is the average SNR of the $\rm R-D$ link.

A. SOP and ASR under Optimal Power Allocation

We first work on the SOP. For secrecy communications, an outage occurs when the secrecy rate $C_s(a)$ is zero. With the optimal power allocation, it happens when $\Gamma_{rd} < 1 + 1/\Gamma_{sr}$. The SOP can thus be derived as

$$\begin{aligned} P_{\rm op}^* &= \int_0^\infty F_{\Gamma_{rd}}\left(1+\frac{1}{t}\right) f_{\Gamma_{sr}}(t)dt \\ &\overset{(a)}{=} 1 - \sum_{k=0}^{N-1}\frac{e^{-\frac{1}{\bar{\gamma}_d}}}{k!\bar{\gamma}_r\bar{\gamma}_d^k}\int_0^\infty\left(1+\frac{1}{t}\right)^k e^{-\frac{1}{\bar{\gamma}_d t}-\frac{t}{\bar{\gamma}_r}}dt \\ &\overset{(b)}{=} 1 - \frac{2e^{-\frac{1}{\bar{\gamma}_d}}}{\sqrt{\bar{\gamma}_r\bar{\gamma}_d}}\sum_{k=0}^{N-1}\sum_{\ell=0}^{k}\frac{\binom{k}{\ell}\left(\frac{\bar{\gamma}_d}{\bar{\gamma}_r}\right)^{\frac{\ell}{2}}K_{\ell-1}\left(\frac{2}{\sqrt{\bar{\gamma}_r\bar{\gamma}_d}}\right)}{k!\bar{\gamma}_d^k}, \end{aligned} \quad (10)$$

where (a) follows from (8) and (9); and (b) follows from the identity $\int_0^\infty x^{-\ell}e^{-a/x-bx}dx = 2\,(a/b)^{\frac{1-k}{2}}K_{\ell-1}(2\sqrt{ab})$ and the binomial expansion. Here, $K_n(\cdot)$ is the modified Bessel function of the second kind.

From (3) and (7), it can be seen that the following are all equivalent to the condition in (5): 1) $\Gamma_{\rm D}^* < 0$, 2) $\Gamma_{\rm R}^* < 0$, and 3) the instantaneous secrecy rate with the optimal power allocation is zero. Thus, the ASR with the optimal power is $\bar{C}_s^* = \mathsf{E}\left\{\log[1+(\Gamma_{\rm D}^*)^+] - \log[1+(\Gamma_{\rm R}^*)^+]\right\}$, i.e.,

$$\bar{C}_s^* = \int_0^\infty \frac{\bar{F}_{(\Gamma_{\rm D}^*)^+}(t)}{(1+t)}dt - \int_0^1 \frac{\bar{F}_{(\Gamma_{\rm R}^*)^+}(t)}{(1+t)}dt, \quad (11)$$

where $\mathsf{E}[\cdot]$ is the expectation and this step comes by using the integration-by-parts method. To evaluate (11), we first derive an exact expression for the c.c.d.f. of $(\Gamma_{\rm D}^*)^+$. For $t \ge 0$,

$$\begin{aligned} &\bar{F}_{(\Gamma_{\rm D}^*)^+}(t) = 1 - \Pr\left[\Gamma_D^* = 0\right] - \Pr\left[0 < \Gamma_D^* \le t\right] \\ &= 1 - \Pr\left[\Gamma_{rd} \le 1 + \frac{1}{\Gamma_{sr}}\right] \\ &\quad - \Pr\left[\Gamma_{rd} > \frac{(2t+1)(1+\Gamma_{sr})}{(\Gamma_{sr}-2)}, \Gamma_{sr} < 2t, \Gamma_{rd} > 1 + \frac{1}{\Gamma_{sr}}\right] \\ &\quad - \Pr\left[\Gamma_{rd} \le \frac{(2t+1)(1+\Gamma_{sr})}{(\Gamma_{sr}-2t)}, \Gamma_{sr} \ge 2t, \Gamma_{rd} > 1 + \frac{1}{\Gamma_{sr}}\right]. \end{aligned}$$

By combining and canceling similar terms with straightforward algebra, we have

$$\bar{F}_{(\Gamma_{\rm D}^*)^+}(t) = \int_{2t}^{\infty} \bar{F}_{\Gamma_{rd}}\left(\frac{(2t+1)(1+x)}{(x-2t)}\right) f_{\Gamma_{sr}}(x)dx \stackrel{(a)}{=} \frac{2(2t+1)e^{-2t(\frac{1}{\bar{\gamma}_r}+\frac{1}{\bar{\gamma}_d})}}{\sqrt{\bar{\gamma}_r\bar{\gamma}_d}e^{\frac{1}{\bar{\gamma}_d}}} \sum_{k=0}^{N-1}\sum_{j=0}^{k} \frac{\binom{k}{j}K_{j-1}\left(\frac{2(2t+1)}{\sqrt{\bar{\gamma}_r\bar{\gamma}_d}}\right)}{k!(2t+1)^{-k}\bar{\gamma}_d^{k-\frac{j}{2}}\bar{\gamma}_r^{\frac{j}{2}}} \quad (12)$$

where (a) follows by first using (8) and (9) in (12), and then applying the transformation $y = x - 2t$. Similarly, the c.c.d.f. of $\bar{F}_{(\Gamma_{\rm R}^*)^+}(t), 0 \le t < 1$, can be derived as

$$\bar{F}_{(\Gamma_{\rm R}^*)^+}(t) = \frac{2(1+t)e^{-\frac{(1+t)}{\bar{\gamma}_d(1-t)}-\frac{2t}{\bar{\gamma}_r(1-t)}}}{\sqrt{\bar{\gamma}_r\bar{\gamma}_d}(1-t)} \sum_{k=0}^{N-1}\sum_{j=0}^{k} \frac{\binom{k}{j}\left(\frac{1+t}{1-t}\right)^k K_{j-1}\left(\frac{2(1+t)}{\sqrt{\bar{\gamma}_r\bar{\gamma}_d}(1-t)}\right)}{k!\bar{\gamma}_d^{k-\frac{j}{2}}\bar{\gamma}_r^{\frac{j}{2}}}. \quad (13)$$

By substituting (12) and (13) into (11), an analytical expression is obtained for the ASR. Unfortunately, it is in an integration form that cannot be further simplified. We thus use numerical integration techniques for its evaluation.

B. Special Scenarios and Discussion

1) High SNR Diversity Analysis: For high SNR, we assume that $\sigma_r^2 = \sigma_d^2 = 1$ and $P = P_s = \delta P_r \gg 1$.

When $N = 1$, by using $xK_1(x) \approx 1$ for small x, we can approximate (10) as $P_{\rm op}^* \approx 1 - e^{-\beta} \approx \mathcal{O}(P^{-1})$, where the SOP decreases with order one.

For $N > 1$, we can lower bound $P_{\rm op}^*$ as

$$P_{\rm op}^* \ge \max\left\{ \Pr(\Gamma_{rd} < 1), \Pr\left(\Gamma_{rd} < \frac{1}{\Gamma_{sr}}\right) \right\} = \begin{cases} \max\{\mathcal{O}(P^{-N}), \frac{\delta(\omega+1)}{P^2(N-1)}\} = \mathcal{O}(P^{-2}); \nu = 0 \\ \max\{\mathcal{O}(P^{-N}), \frac{\delta(\omega P/\delta+1)}{P^2(N-1)}\} = \mathcal{O}(P^{-1}); \nu = 1 \end{cases} \quad (14)$$

where the last approximation comes for high P as: i) $F_W(1) \approx (\beta)^N/N!$; and ii) the integration can be solved by using $F_Z(t) = \sum_{k=1}^{\infty}(-t/\bar{\gamma}_r)^{k+1}/k! \approx t/\bar{\gamma}_r$ for $\bar{\gamma}_r \gg 1$, and $f_W(t)$ in (8). Further, the SOP can be upper bounded by

$$P_{\rm op}^* \le \Pr(\Gamma_{rd} < 2) + \Pr\left(\Gamma_{rd} < \frac{2}{\Gamma_{sr}}\right),$$

from which the same diversity order as in (14) can be obtained. Thus the outage probability decreases with order one and two for the two interference models $\sigma_i^2 = \omega P_r$ and $\sigma_i^2 = \omega$, respectively. Having $N \ge 2$ does not have significant impact on the outage at high SNR (which will be shown in Fig. 2c).

2) ASR for Single-Antenna Destination: We now consider the special case of single-antenna destination where $N = 1$. The c.c.d.f. $\bar{F}_{\Gamma_{\rm D}^*}(t)$ and $\bar{F}_{\Gamma_{\rm R}^*}(t)$ can be approximated as

$$\bar{F}_{(\Gamma_{\rm D}^*)^+}(t) \approx e^{-\frac{1}{\bar{\gamma}_d}-2t(\frac{1}{\bar{\gamma}_r}+\frac{1}{\bar{\gamma}_d})}, \; \bar{F}_{(\Gamma_{\rm R}^*)^+}(t) \approx e^{-\frac{(1+t)}{\bar{\gamma}_d(1-t)}-\frac{2t}{\bar{\gamma}_r(1-t)}}$$

for $t \ge 0$ by using $xK_1(x) \approx 1$ for small x. The approximations are tight for high SNR/SINR. By using these approximations and (11), we have

$$\bar{C}_s^* \approx e^{\frac{1}{\bar{\gamma}_r}}\mathrm{Ei}\left(-\frac{1}{\bar{\gamma}_r}-\frac{1}{\bar{\gamma}_d}\right) - 2e^{\frac{2}{\bar{\gamma}_r}+\frac{1}{\bar{\gamma}_d}}\mathrm{Ei}\left(-\frac{2}{\bar{\gamma}_r}-\frac{2}{\bar{\gamma}_d}\right) = \begin{cases} \ln(P) - \gamma + \mathcal{O}\left(\frac{1}{P}\right); & \nu = 0 \\ e^{\frac{\omega}{\delta}}\mathrm{Ei}(\frac{-\omega}{\delta}) - 2e^{\frac{2\omega}{\delta}}\mathrm{Ei}(\frac{-2\omega}{\delta}) + \mathcal{O}\left(\frac{1}{P}\right); & \nu = 1 \end{cases} \quad (15)$$

where $\mathrm{Ei}(\cdot)$ is the exponential integral function, γ is the Euler's constant, and we use $\sigma_r^2 = \sigma_d^2 = 1$ and $P = P_s = \delta P_r \gg 1$. This shows that, for high SNR, the ASR increases logarithmically with transmit power for the RSI model where $\sigma_i^2 = \omega$, and has an ASR floor for the RSI model where $\sigma_i^2 = \omega P_r$.

3) Large-Antenna Destination: For $N \to \infty$, the $\mathrm{R}-\mathrm{D}$ channel Γ_{rd} is said to provide asymptotic channel hardening as $\Gamma_{rd}/N \to \bar{\gamma}_d$ almost surely. The instantaneous channel gain converges to the deterministic average channel gain. Thus, for this asymptotic analysis, we use $\Gamma_{rd} \approx N\bar{\gamma}_d$. For $N\bar{\gamma}_d < 1 + 1/\Gamma_{sr}$ and given $N\bar{\gamma}_d > 1$, the SOP under the optimal power allocation is then derived as

$$P_{\rm op}^* = 1 - e^{-\frac{1}{\bar{\gamma}_r(N\bar{\gamma}_d-1)}} \approx \frac{1}{\bar{\gamma}_r\bar{\gamma}_d N} + \mathcal{O}\left(\frac{1}{N^2}\right), \quad (16)$$

which is inversely proportional to N.
To evaluate the ASR, we can derive exact expressions for the c.c.d.f.s of $(\Gamma_{\rm D}^*)^+$ for $t \ge 0$ and $(\Gamma_{\rm R}^*)^+$ for $0 \le t < 1$, respectively, as

$$\bar{F}_{(\Gamma_{\rm D}^*)^+}(t) = \bar{F}_{\Gamma_{sr}}\left(\frac{2t(N\bar{\gamma}_d+1)+1}{N\bar{\gamma}_d-1-2t}\right); \qquad t \le \frac{N\bar{\gamma}_d-1}{2},$$

$$\bar{F}_{(\Gamma_{\rm R}^*)^+}(t) = \bar{F}_{\Gamma_{sr}}\left(\frac{t(2N\bar{\gamma}_d+1)+1}{(N\bar{\gamma}_d-1)-t(N\bar{\gamma}_d+1)}\right); t \le \frac{N\bar{\gamma}_d-1}{N\bar{\gamma}_d+1},$$

where $\bar{F}_{\Gamma_{sr}}(t) = e^{-t/\bar{\gamma}_r}$ and c.c.d.f. values are zero for other t values. With the aid of (11) and further mathematical manipulations, the ASR can be derived as

$$\bar{C}_s^* = e^{\frac{1}{\bar{\gamma}_r}}\left[e^{\frac{w}{\bar{\gamma}_r}}\mathrm{Ei}\left(\frac{w^2}{\bar{\gamma}_r(1-w)}\right) - 2e^{\frac{w}{\bar{\gamma}_r(w+1)}}\mathrm{Ei}\left(\frac{2w^2}{\bar{\gamma}_r(1-w^2)}\right) + \mathrm{Ei}\left(\frac{w}{\bar{\gamma}_r(1-w)}\right)\right] \quad (17)$$

$$\approx e^{\frac{1}{\bar{\gamma}_r}}\mathrm{Ei}\left(-\frac{1}{\bar{\gamma}_r}\right) - 2e^{\frac{2}{\bar{\gamma}_r}}\mathrm{Ei}\left(-\frac{2}{\bar{\gamma}_r}\right) + \mathcal{O}\left(\frac{1}{N}\right), \quad (18)$$

which is independent from N and statistic of $R-D$ channel.

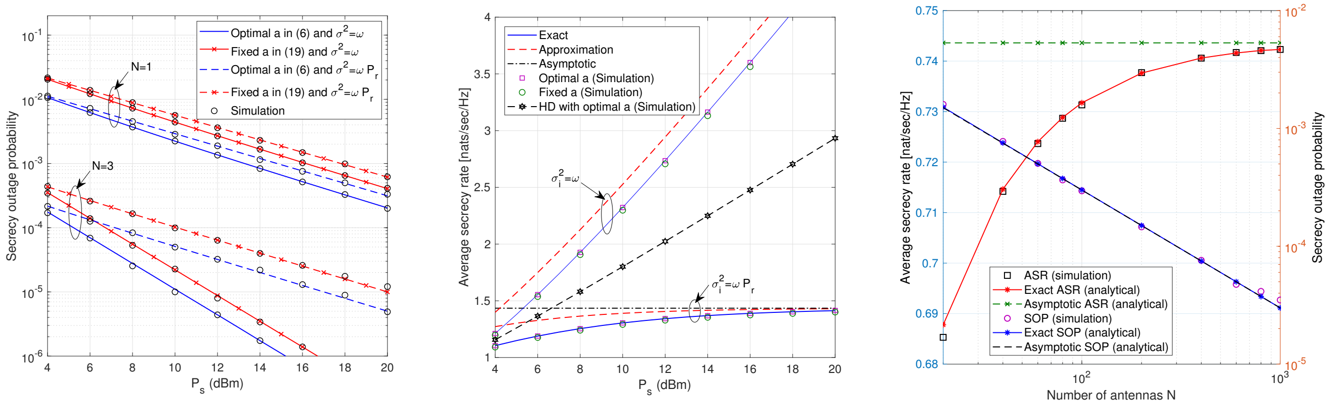
4) Power Allocation Independent of Instantaneous Channel Values: The optimal power allocation depends on the instantaneous channel values. If instantaneous CSI is unavailable we may select a based on the statistical information of channels. For an arbitrary but constant a, the event $C_s(a) = 0$ occurs when $\gamma_{\rm D} \le \gamma_{\rm R}$, equivalently, $(1-a)\Gamma_{rd} < (1 + 1/\Gamma_{sr})$. The outage probability can then be derived as

$$P_{\rm op}(a) = \Pr\left[\Gamma_{rd} < \frac{1+\frac{1}{\Gamma_{sr}}}{1-a}\right] = P_{\rm op}^*\Big|_{\bar{\gamma}_d=(1-a)\bar{\gamma}_d}, \quad (19)$$

which means that the outage probability can be obtained from that of the optimal power allocation case by replacing $\bar{\gamma}_d$ in (10) with $(1-a)\bar{\gamma}_d$. When instantaneous CSI is unavailable, according to (6), a reasonable choice of a is

$$\hat{a} = \frac{1}{2}\left(1 - \frac{\bar{\gamma}_r+1}{\bar{\gamma}_r\bar{\gamma}_d}\right), \quad (20)$$

whose SOP can be calculated from (19) and (10). However, the ASR achieved by $\hat{a}$ cannot be derived similarly from the result of the optimal power allocation case. The reason is that the SINR expressions $\Gamma_{\rm R}(\hat{a})$ and $\Gamma_{\rm D}(\hat{a})$ in (1) do not have the advantageous structure as in (7) (where $\Gamma_{\rm R}(a^*)$ and $\Gamma_{\rm D}(a^*)$ as have the same numerator).



(a) SOP vs P_s for optimal a^* and fixed $\hat{a}$.

(b) ASR vs P_s for $N=1$.

(c) Performance of large-antenna destination.

Fig. 2: The SOP/ASR vs P_s when $P_r = P_s/2$ for different scenarios.

V. Numerical Results

We set channel variances $\sigma_{hs}^2 = \sigma_{rd_\ell}^2 = 1$, RSI parameter $\omega = 0.1$, noise variances $\sigma_r^2 = \sigma_d^2 = 0.01$ and $P_r = P_s/2$.

Fig. 2a shows the SOP vs P_s for different cases. Several observations are gained: i) Our analytical results closely match with the simulation results, which confirms the accuracy of our analysis; ii) The optimal power allocation in (6) always outperforms the fixed allocation in (20) where we achieve around 3.5 dB gain with $N=3$ at 10 dBm; iii) The case with $\sigma_i^2 = \omega$ outperforms the case with $\sigma_i^2 = \omega P_r$ because self-interference with $\nu = 1$ increases with P_r which decreases SINR at D; and iv) For $N=1$, the SOP decreases with P_s of order one for both $\nu = 0$ and $\nu = 1$. For $N=3$, the SOP decreases with P_s of order one and two for $\nu = 1$ and $\nu = 0$, respectively. This confirms the results in Section IV-B, and shows the benefit of having multiple antennas at D. Fig. 2b shows the ASR vs P_s for $N=1$. While exact analytical results are evaluated numerically by using (11), (12) and (13), the approximation and asymptotic results are obtained by using (15). The ASR increases linearly with log-scale P_s for $\sigma_i^2 = \omega$ and has an ASR floor for $\sigma_i^2 = \omega P_r$ which confirms the result in (15), e.g., the ASR gap is around 2 [nats/sec/Hz] (or 4 dB gain) at 15 dBm. Also, the fixed $\hat{a}$ has very close ASR as the optimal a^* with negligible gap at moderate P_s. Further, FD relaying outperforms the HD relaying when $\sigma_i^2 = \omega$. Fig. 2c shows the ASR vs N and SOP vs N for a large-antenna destination with $P_s = 0$ dBm, $\sigma_i^2 = 0.1$ and 0.1 noise variance. Numerical results are evaluated by using (16), (17) and (18). Simulation and analytical results match tightly. The ASR gradually increases with N and approaches its asymptotic value 0.74 which is determined by (18). The SOP decreases with order one of N as shown in (16).

VI. Conclusion

The SBJ is considered for a network with a source, an FD-AF untrusted relay and a multiple-antenna destination. We first derive the optimal power allocation factor between intended and jamming signals at the source. For the optimal case, we derive i) the exact SOP; ii) asymptotic SOP expressions and iii) distributions of optimal SINRs at the relay and destination in order to evaluate the average secrecy rate. For a single-antenna destination, we provide approximation and asymptotic expressions. We then discuss the applications of these results for fixed-power allocation and large-antenna destination. It is revealed that the impact of having $N \geq 2$ on the outage is not significant. By suppressing self-interference, as it is independent of transmit power, FD achieves higher secrecy rate than HD relaying.

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